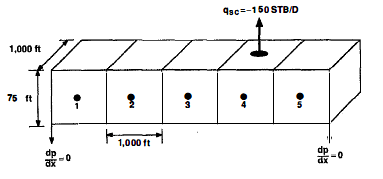
# Appendix D - Analytical Solution to 1D Single-phase Fluid Flow



For the 1D, block-centered grid shown above, determine the pressure distribution during the first year of production. The initial reservoir pressure is . The rock and fluid properties for this problem are , , , , , , , , and .

Consider the following equation (as derived in **App. A**) with its initial and boundary condition

|  |  |
| --- | --- |
| Partial differential equation (PDE) | |
|  | |
| Initial condition (IC) | Boundary condition (BC) |
|  |  |
| Other specification |  |
|  |  |

First, we need to assume that the only variable that changes with respect to and is . All else are constants. We can group variable and define it as . Similarly, we define as . The PDE then becomes

Recognize that the equation is a non-homogeneous PDE of the form , with being constant for every and . We can apply **Laplace transform** in this case. Recall the following definition of Laplace transform for some function of and

*Remark*. We are performing Laplace transform **with respect to** .

Applying that to the PDE

For clarity, let denote

Similarly, perform Laplace transform to BC and IC. We are now given the following specification

|  |  |
| --- | --- |
| Partial differential equation (PDE) | |
|  | |
| Initial condition (IC) | Boundary condition (BC) |
|  |  |

Now we have an ordinary differential equation (ODE) with that varies only with respect to . It is still a non-homogeneous ODE however. We need to break down its form into homogeneous and particular form as follows

It follows that

Suppose has a solution of the form

The characteristic equation is . The roots are and . Therefore

As for , we use **method of undetermined coefficients**. We take a solution of the polynomial form, .

It follows that

We obtain the general form of , which is

Now we need to look into . The behavior of is still unknown. Since it will affect the variable , i.e. if , we will get an imaginary number.

1. **Assuming**

We need to verify the general form of by evaluating its boundary condition

In order to obtain , we need to perform inverse transform

The above general expression satisfies the initial condition. But we still need to determine coefficient by cross-checking it with boundary condition.

We get a trivial solution.

1. **Assuming .**

By **Euler’s formula**, we can evaluate any term as described by the following formula

Since is some negative number, we can rewrite it as . Our earlier general expression of can then be rewritten into the following,

**to be continued..**